NCSL National College for School Leadership

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Development tool

What are we
learning about...?

'Making mathematics count' in school networks

Multiplying the learning through collaborative enquiry

Enquiring together in mathematics

A development tool designed to provide a framework for exploring the processes and outcomes of collaborative enquiry in mathematics in order to support the transfer of learning to others.

Networked Learning Communities

learning from each other | learning with each other | learning on behalf of each other

Multiplying the learning through collaborative enquiry

This development tool is designed to provide a framework for exploring the processes and outcomes of a collaborative enquiry undertaken by a network of mathematics practitioners. It gives illustrative guidance on writing up an enquiry summary to share with others, with the aim of multiplying the learning from the enquiry through dissemination. Although there are many methods of disseminating research findings, the most common remains writing up the enquiry project. The illustrative example of collaborative enquiry drawn upon throughout this development tool is presented in a tried and tested format (see opposite) originally devised by the Centre for the Use of Research and Evidence in Education (*Writing research and enquiry summaries*, NCSL & CUREE, 2005).

The key features of collaborative enquiry are that it should be a voluntary activity undertaken to improve learners' experiences. In order to establish a sense of shared ownership, the enquiry starts with a question identified through the shared expertise of the practitioners. With strong institutional support, they then build their team. They share responsibility for interpreting the agreed focus and develop a plan, structure and timetable. The significance of belonging to something important and purposeful can be greatly enhanced by recruiting the services of other experts such as local Higher Education or Local Authority staff. It is critically important that the team accords equal respect to everybody's ideas and is committed to disseminating its findings to other practitioners – so maximising the potential for multiplying the learning from the enquiry, by informing the practice of others.

By drawing upon an illustrative example of collaborative enquiry in action and utilising the framework provided by the building blocks for writing up an enquiry summary (detailed opposite), this development tool provides a template (see pages 9-11) for use in supporting mathematics practitioners in conducting a collaborative enquiry and sharing their learning with others. □

Acknowledgements

We would like to thank Paul Andrews and Barbara Ball for their contribution to the generation and publication of this development tool.

Nick Martin and Non Worrall
Lead Developers

1 Title
Include key words to tune the reader into the focus of your enquiry.

2 Identification
Include yourself and your network, keywords, levels of learning, intended audience.

3 Aim(s) of the enquiry
What problems or issues did the study tackle? What questions formed the basis of your enquiry?

4 Dimensions of the study
Gather concise information about the who, what, where and when aspects.

5 Summary of main findings
What were the headline findings? Is there any evidence of positive impact on teachers and pupils?

6 Background and context
Why and how did the project start? How does it build on existing knowledge? Who was involved?

7 The intervention: process and strategies
What actually happened in the school or classroom? Examples of strategies and how they were implemented are useful. Mention pitfalls and how they were overcome.

8 The detailed findings
Explain each of the findings in more depth. What evidence is there to support these findings?

9 Enquiry methods
What sort of data were collected? How were they collected and analysed?

10 Conclusions
What are your thoughts about the study? How have the findings affected your school or the way you do things now? Have they raised any further questions for investigation?

11 Further reading
How was your work linked with what is already known about the theme of your enquiry? What literature could you recommend to teachers to find out more?

12 The detailed findings
Who could teachers contact if they want to find out more?

Enquiry summary

1 Title
Include key words to tune the reader into the focus of your enquiry.

Developing learners' awareness of mathematical thinking

2 Identification
Include yourself and your network, keywords, levels of learning, intended audience.

Author
Barbara Ball, Association of Teachers of Mathematics

Network
5 Leicester schools

Keywords
mathematics, problem-solving, collaborative enquiry

Levels of learning
pupil learning, adult learning, leadership learning

Audience
teachers of mathematics, HEI partners, Local Authority officers

3 Aim(s) of the enquiry
What problems or issues did the study tackle? What questions formed the basis of your enquiry?

How can teachers help students to develop their mathematical thinking?

- The teachers themselves worked collaboratively on mathematical problems.
- The teachers paid attention to awareness of themselves as mathematical problem solvers.

4 Dimensions of the study
Gather concise information about the who, what, where and when aspects.

Mathematics teachers from five 11–16 Leicester schools undertook the research between January and June 2004. They worked as a collaborative group, supported by an academic leader and educational consultant. Each teacher chose a particular Key Stage 3 class to focus on, to try out the mathematical thinking strategies they devised.

5 Summary of main findings
What were the headline findings? Is there any evidence of positive impact on teachers and pupils?

Pupil and teacher outcomes

- Useful strategies were identified to promote mathematical thinking.
- Teachers became more relaxed and confident.
- Teachers enabled learners to take increasing responsibility for the management of discussion in the lesson.
- Improved questioning created productive and co-operative classrooms.
- Students became more engaged and better behaved.
- Students could articulate publicly their ever-developing skills of mathematical thinking.
- Students demonstrated significant cognitive development beyond what normally might have been expected.

“ The enquiry has made me think carefully about how I set tasks, how much I say, what I say and the effects it has on students' learning if you get it right! ”

Points raised about learning networks of mathematics teachers

Principles underpinning the enquiry:

- Participating teachers had well-developed and considered beliefs about teaching and learning.
- Professional development as a process should acknowledge the context and aspirations of participating teachers.
- Professional development takes place over time and should provide adequate opportunities for trial of and reflection on ideas and activities.
- Professional development activities should facilitate the creation of learning networks.

“ From our discussions, I have seen how effectively leading students through and/or demonstrating solving a problem can be as a means of helping them acquire thinking/ problem solving strategies. I no longer shy away from this. ”

Background and intervention

6 Background and context
Why and how did the project start? How does it build on existing knowledge? Who was involved?

The background to the enquiry

The starting point for this enquiry was a funded project which involved the Association of Teachers of Mathematics and the Mathematical Association working with a small group of teachers, drawn from different challenging inner-city schools, on developing teaching strategies that encourage learners to acquire high-level mathematical reasoning skills.

The project was based on the use of a collaborative enquiry which adopted a bottom-up rather than top-down approach, in order to build on teachers' expertise and interests.

The focus for enquiry

The focus on mathematical thinking was chosen by the subject associations as this was a facet of mathematics teaching identified by Ofsted and the National Strategies that needed development. The initial proposal derived from work done by an Advanced Skills Teacher (AST) as part of her Best Practice Research Scholarship (BPRS) on meta-cognition (Flavell, 1976).

The proposal suggested that one way to help teachers develop their students' mathematical thinking was for the teachers to work collaboratively on mathematical problems and to pay attention to their own awareness of themselves as mathematical problem solvers.

The structure of the network

The enquiry team was led by an academic who was supported by a teacher mentor and the AST whose BPRS report provided the initial focus for the enquiry. The Local Authority adviser for Leicester negotiated the involvement of five teachers whom she thought would be most likely to benefit from, and be willing to commit themselves to, the enquiry. One was a head of department, three held posts of responsibility within their departments and one was in her second year of teaching.

7 The intervention: process and strategies
What actually happened in the school or classroom? Examples of strategies and how they were implemented are useful. Mention pitfalls and how they were overcome.

A two day residential conference offered a rationale for the enquiry and included a wide range of activities intended to:

- facilitate the teachers getting to know each other and the enquiry team
- focus the teachers' attention on to their awareness of their own thinking when doing mathematics collaboratively
- provide the teachers with the opportunity to consider how the mathematical activities offered might be used in their own classrooms

The academic leader videoed each teacher working with the class the teacher had chosen to focus on in the enquiry. At a one day follow-up meeting the group viewed and discussed the videos. Two half day meetings were then used to review progress and to plan the next stage of the enquiry. During this time the teacher mentor visited each teacher and both taught and observed a lesson, the academic leader made a second video and the teachers visited each other's classrooms. A final one day conference reviewed the effects of the enquiry.

Classroom activities

During the first conference, time was spent discussing the crucial role of the teacher in helping students to develop their own mathematical thinking. In particular, the group discussed the influence of the way in which the task is set up on the subsequent learning. Discussion also took place about the importance of selecting the right type of questions to ask students. A significant proportion of the conference was spent in working collaboratively on mathematical tasks that could then be used by the teachers with their selected classes. The following questions were used to inform this process.

Questions for classroom activities

- How did you set up the task for the learners?
- How open was the task?
- How did your class respond to the task and was the degree of openness appropriate?
- What would you do differently next time and why?
- What were the outcomes with respect to the students' mathematical thinking?
- What evidence do you have to support your conjectures?
- What was your role in making the activity successful or otherwise?
- Were your interventions more successful in whole-class working or in individual working? Why?

Detailed findings and methods

8 The detailed findings
Explain each of the findings in more depth. What evidence is there to support these findings?

As a result of watching the videos a number of different strategies that teachers had found useful were identified:

- explain
- think aloud
- give instructions
- say less – most of the teachers commented while watching their videos that they talked too much.
- "I don't know the answer, how can I find it?"
- "here's the answer, how did I get it?"
- "you've got a minute to do …"
- "convince me"

One teacher said she was going to stick a copy of these strategies on her desk and it was agreed that the teachers would keep the strategies and the following questions in mind when they wrote their journals:

- What strategies have you tried with your classes?
- What influenced your decision to try them?
- What was your perception of their effectiveness?
- How would you use them in the future?

Further useful strategies from the enquiry

- Inviting students to 'convince me' or 'convince a friend'.
- Giving students the answer and inviting them to find the method of obtaining it.
- Inviting students to explain to each other.
- Alerting students to the fact that you will be making deliberate mistakes.
- Using individual whiteboards or scrap paper, so that incorrect working is seen not to matter – the use of disposable answers helps students gain confidence in what they do.
- Inviting other students to explain or tell – rather than the teacher – when students have problems.
- Using things like 'what?' And 'why?' Cards, on which students write what they have found out and why their approach worked.
- Encouraging students to create posters and displays.

9 Enquiry methods
What sort of data were collected? How were they collected and analysed?

Teachers' journals

The teachers kept journals in which they reflected on particular lessons throughout the project which recorded:

- What the teachers planned to do, how they planned to do it and why.
- A factual, or as factual as they could make it, account of what happened when they did it.
- A sense of the teachers' emotional and intellectual responses to the outcomes.
- A sense of the teachers' awareness and understanding of any changes in their own or their students' behaviour.

The quotations used to illustrate this account are taken from the teachers' journals.

"The introduction of smaller problems with mathematical content at a lower level gave all students in the group the chance to experience success. This was the real impact for change. Their approach to other lessons is now more 'I can' than 'what do I have to do?'"

Videos of teachers

It was agreed at the first meeting that each teacher would be videotaped twice working with the designated class. The first recording would be made near the beginning of the enquiry and the second near the end. The intention was to facilitate teachers' reflections on their professional activity and to enable them to detect ways in which their students' articulation of mathematical thinking developed during the short lifetime of the enquiry.

At the second meeting of the group the videos were viewed and discussed by the whole team. Each teacher identified two clips from the lesson: one which showed an episode in which the teacher acted to help learners think mathematically, and one which showed an episode in which the teacher's action failed to help learners to think mathematically.

"Shared observation was very rewarding, an opportunity to discuss my teaching strategies on common ground and compare these with the mentor's strategies. This led to a lot of 'I wonder ifs', which I can explore."

Conclusions

10 Conclusions
What are your thoughts about the study? How have the findings affected your school or the way you do things now? Have they raised any further questions for investigation?

Evaluating the enquiry

During the last month, teachers were asked to reflect on what they had done by answering these questions:

- What impact has the project had on how you teach?
- How has this change come about?
- What do you think has changed most?
- How confident are you in these changes and why do you feel this way?
- What do you think may happen in the coming months?
- How have your students changed during this enquiry?
- What changes have you noticed in their behaviour?
- What changes have you noticed in their mathematical thinking?
- Has the enquiry had any impact, for example, on students' attitudes?
- What would you expect to be the long-term implications and why?

The impact of the enquiry on learners

Teacher expectations

It was clear that the teachers were frequently surprised by their students appearing to be able to manage work or ideas conventionally thought to be too difficult for them. Teacher F, when discussing the responses of equivalent classes to similar tasks commented: *"the difference in how the students accessed the problems and how they developed their reasoning was very noticeable. The students actually took significantly less time to get to the answers, so much so that the lesson finished 20 minutes earlier than I had anticipated".*

Student attitudes

All teachers made some allusion to the impact of the enquiry on student attitudes. Several commented on perceived changes in classroom atmosphere and of a growing sense of collaboration. Throughout Teacher J's account, one finds expressions like the *"class worked enthusiastically", "were eager to continue"* and *"in an exhilarating 15 mins or so most students felt a real sense of ownership as they proved what they had already suspected".*

Teacher B made the important point that *"some of the students who really responded well to solving problems and explaining their thinking were those who would tend to cause behaviour problems in some of their lessons. It might be that this approach motivates these students".*

Students' thinking

Teacher F offered a number of insights into the impact of the enquiry on the thinking of several of her Year 7 class, perceived by the school as of low ability (set 5 out of 6 and working towards level 4). She made several comments indicative of shifts in their cognition. In describing a task related to finding triangles on a three-by-three grid she commented that *"all students engaged with this"* and there was a *"lot of success with vocabulary and with all wanting to argue with each other".* In terms of another activity she commented that *"the discussion here became very interesting and the students wanted to be heard and almost seemed to be less afraid of errors and more accepting of the explanations that their peers were offering".*

Her comments about a problem concerning consecutive numbers included:

> ***"There were a number of students who actually surprised me, pleasantly, with their willingness to be left alone and to discuss within their groups. My anticipation of some students struggling with the open-ended structure was justified, but not necessarily with the students I had thought!"***

Teacher E's comments related to a bottom-set Year 8 class, reflected a growing confidence in her learners' ability to articulate their thinking. Fairly early in the project she noted that her *"pupils were too scared to try something of their own"* with the consequence that she *"ended up making suggestions".* A number of lessons later, she commented that a task involving students coming to the board, placing a statement on a probability line and justifying their decisions was effective and *"caused debates and got pupils to consider all aspects of a statement".*

In summary, the comments recorded by project teachers seemed to indicate a perception, derived from their reflections on their lessons, that their students were becoming more confident in articulating mathematical reasoning in a public forum. Such behaviour accords with that found in mathematics classrooms in countries such as Hungary where high-level thinking is encouraged and where mathematics attainment is generally high (Andrews, 2003).

Students' attainment

In four cases out of five, teachers were able to provide data which compared the performance in end-of-year tests of the class they had focused on during the enquiry with that of a similar class. Because the control class was taught by a different teacher, the results have to be treated with caution. However, the assessment evidence of all four teachers seems to indicate that levels of student achievement were higher than might normally have been expected.

This raises the question of what is producing these differences. It is impossible to assert that such differences are due to the work of the enquiry, after all, it lasted only six months. However, other evidence would suggest that the enquiry has had an impact on how teachers work with, and how students respond to, mathematics.

Indeed, one of the most interesting and reassuring issues to emerge from the data was that several colleagues commented that they were worried that the enquiry emphases – on mathematical thinking and reasoning rather than explicit curriculum content – might militate against their students' assessment performance. This was clearly not the case and presents a powerful message in respect of what teachers might be expected to do in their classrooms.

Students' behaviour

The video evidence supported the teachers' view that students' behaviour had changed during the course of the enquiry. Students were making more thoughtful contributions, they were listening better and discussing their work in more mature ways than at the outset. In some cases, relative to their age and purported developmental levels, they were offering sophisticated, contributions to the mathematical discourse of their classes.

Such contributions were built on by increasingly confident teachers who were beginning to feel at ease with new ways of working or being allowed to reintegrate practices they felt had been marginalised by current curricular and assessment practices.

The impact of the enquiry on teachers

One of the abiding findings to emerge was that the teachers felt that the enquiry had given them an authority to work outside perceived constraints of imposed frameworks. This led to a sense of regained empowerment on the part of both teachers and their learners, who were now experiencing a form of mathematics different from that with which they were familiar. Teacher J commented that *"we used to do a lot of stuff 10 or 15 years ago but somehow it has got squeezed out by the national curriculum and other initiatives"*.

The teachers' journals reflect a clear sense of growth and a regaining of professional confidence. Teacher E noted in her journal some early failures, for example in attempting to avoid giving too much away when posing a problem concerned with shapes on a dotty grid, she noted that her *"pupils were too scared to try something of their own, so I ended up making suggestions, for example to try some more shapes with no dots inside"*. But, interestingly, the same strategy was described as *"very effective"* when used six weeks later once her students had begun to acquire confidence in themselves and their abilities to construct both new avenues and justifications for what they did. This sense of changing approaches to teaching was apparent in the comments of all colleagues.

Several teachers commented on the need to structure their learners' developing skills. It became apparent early in the project's work that too open a task would lead to learner uncertainty and confusion.

> ***"The most significant change would be the difficulty in actually accessing the problem, making students realise the need to rethink their existing strategies for problem solving. Old methods are not the only methods. They would need to learn how to process the information given, which parts are appropriate and how their understanding can be used to answer the challenge. My intention was for students to be exposed to a variety of strategies that have come from me, themselves and each other. This way they would be able to select a method that works for them and that fits the problem itself."***

Inevitably, not all the teachers' strategies proved successful. Teacher J commented that she had offered a grid method for solving logic problems which was *"enthusiastically welcomed by some but seen as an unnecessary complication by others"*. Teacher E wrote of several episodes in which her attempts had not been as successful as she would have wished. However, none of the teachers was deterred by temporary setbacks. For all of them, it was a learning process with considerable gains.

Further reading and follow-up

11 Further reading
How was your work linked with what is already known about the theme of your enquiry? What literature could you recommend to teachers to find out more?

Andrews, P, and Hatch, G, 2000, *A comparison of Hungarian and English teachers' conceptions of mathematics and its teaching.* Educational Studies in Mathematics, 43 (1), pp 31–64

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12 The detailed findings
Who could teachers contact if they want to find out more?

This article by Barbara Ball, Professional Officer of the Association of Teachers of Mathematics, is based on the draft final report of the enquiry written by Paul Andrews, Senior Lecturer in Mathematics Education at the University of Cambridge.

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References

If you would like to obtain a copy of the booklet *Writing research and enquiry summaries* (NCSL & CUREE, 2005) which provides detailed guidance on the building blocks for writing up collaborative enquiry utilised throughout this development tool – please email **nlc@ncsl.org.uk** or alternatively download directly from **www.nlcexchange.org.uk**

Collaborative enquiry summary template

Use this template to write up a summary of your collaborative enquiry to share with others.

Title

Include key words to tune the reader into the focus of your enquiry.

Identification

Include yourself and your network, keywords, levels of learning, intended audience.

Aim(s) of the enquiry

What problems or issues did the study tackle? What actual questions were researched?

Dimensions of the study

Concise information about the who, what, where and when aspects.

Summary of main findings

What were the headline findings? Is there any evidence of positive impact on teachers and pupils?

Collaborative enquiry summary template

Background and context

Why and how did the project start? How does it build on existing knowledge? Include relevant details about the school and network context and the pupils and teachers involved.

The intervention: process and strategies

What actually happened in the school or classroom? Examples of strategies and how they were implemented are useful. Mention pitfalls and how they were overcome.

The detailed findings

Explain each of the findings in more depth. What evidence is there to support these findings?

Enquiry methods

What sort of data were collected? How were they collected and analysed?

Conclusions

What are your thoughts about the study? How have the findings affected your school or the way you do things now? Have they raised any further questions for investigation?

Further reading

How was your work linked with what is already known about the issues you were researching? What literature could you recommend to teachers to find out more?

The detailed findings

Who could teachers contact if they want to find out more?

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The 'What are we learning about…?' series is designed to make public the learning that has emerged from NLCs in the last two years.

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- Leadership of school networks

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